EXAMINATION OF TRANSIENT EVENTS

PREPARATION QUESTIONS

1. What is transient event and steady event? Explain briefly.
2. Determine the current-voltage relations of capacitor and inductance elements.
3. Determine the equations for the time-dependent change of the voltage at the terminals of a capacitor as it charges and discharges through a resistor together with its graphs.
4. Explain the solution steps of second-order homogeneous differential equations with constant coefficients.
5. Examine the time variation of the voltage at the terminals of a capacitor charged with voltage E as it discharges through R and L, connected in series with each other.
6. Examine the features of a breadboard and learn how to set up an electrical circuit on a breadboard.

NOTE: Come to the experiments by preparing the preparation work as a report (with the report cover). Those without a preparation report will not be admitted to the experiments.

EXPLANATIONS

The behavior of systems when they pass from one steady state to a second steady state is called transient events. This experiment will analyze transient events in an electrical circuit consisting of R, L, and C elements.

In the circuit in Figure 1, in the first continuous state the switch is in position 1, and in this state, capacitor C is charged with voltage E. Therefore, i=0, U_C =E, U_R=0, and U_L=0.

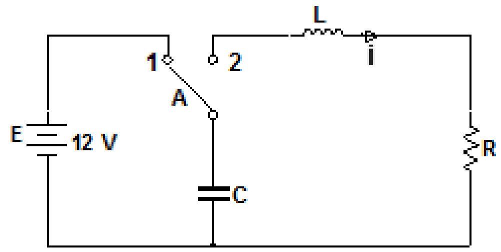

Figure 1

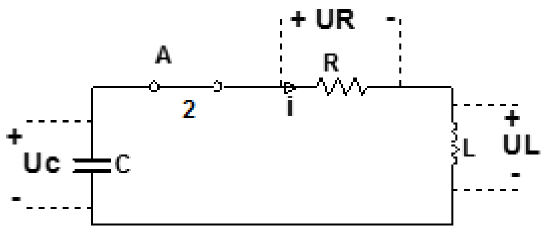

Figure 2

The second steady state is reached after a certain time after switch A is turned to position 2. In this state, i=0, U_C =0, U_R=0, and U_L=0, i.e., the capacitor which was filled up to voltage E in the first state will be completely discharged through R and L after the switch is turned to position 2, and the second steady state will be reached. Now let us examine the transient events between these two steady states. When the switch is turned to position 2, the circuit will be formed in Figure 2. In this circuit, we can obtain equations (1) and (2) with a peripheral equation written to obtain the Uc voltage;

$$U_R + U_L - U_C = 0 \qquad (1)$$

$$R.i + L \cdot \frac{di}{dt} + \frac{1}{C}\int idt = 0 \qquad (2)$$

Using equations (1) and (2), we can find the variation of the current circulating in the system and the voltage at the terminals of the circuit elements with respect to time. For example, let's determine the voltage change at the terminals of the capacitor. Relation between capacitor voltage and current;

$$i = -C\frac{du_C}{dt} \tag{3}$$

if we substitute the current expression i in equation (2);

$$RC\frac{du_C}{dt} + LC\frac{d^2u_C}{dt^2} + u_C = 0 \tag{4}$$

$$\frac{d^2u_C}{dt^2} + \frac{R}{L}\cdot\frac{du_C}{dt} + \frac{1}{LC}u_C = 0 \tag{5}$$

equations (4) and (5) are obtained. If we denote the coefficients of the equation by D=R/L (D is the damping constant) and w_0^2 = 1/LC (w_0 is the eigenfrequency of the circuit), the differential equation determining the variation of the capacitor voltage is found as equation (6).

$$\frac{d^2u_C}{dt^2} + D\frac{du_C}{dt} + w_0^2u_C = 0 \tag{6}$$

As is known, to solve such equations, the characteristic equation of the differential equation must first be written. The characteristic equation for equation (6) is given in (7):

$$p^2 + D.p + w_0^2 = 0 \tag{7}$$

from the equation, the roots of equation (7):

$$p_{1,2} = -\frac{D}{2} \pm \sqrt{\left(\frac{D^2}{4} - w_0^2\right)} \tag{8}$$

Depending on the value of the roots obtained by equation (8), the instantaneous value of the voltage at the terminals of the capacitor when discharging is given by equation (9).

$$U_C(t) = k_1 \cdot e^{p1t} + k_2 \cdot e^{p2t} \tag{9}$$

The constants k_1 and k_2 determine the initial state of the circuit. When the switch was in position 1, capacitor C was charged with voltage E. Then, when the switch is turned to position 2 (t=0), the capacitor voltage will be as shown in equation (10);

$$U_C(t) = k_1 \cdot e^{p1t} + k_2 \cdot e^{p2t} \tag{10}$$

The current flowing through the circuit is found as in equation (11);

$$i = -C\frac{du_C}{dt} = -C(k_1p_1e^{p_1t} + k_2p_2e^{p_2t}) \tag{11}$$

Since the current cannot jump due to the inductance effect when the switch is closed, equation (12) is found.

$$i(0) = 0 = k_1 \cdot p_1 + k_2 \cdot p_2 \tag{12}$$

Using relations (10) and (12), the constants k_1 and k_2 are found as in equation (13):

$$k_1 = E\frac{p_2}{p_2 - p_1} \text{ and } k_2 = -E\frac{p_1}{p_2 - p_1} \tag{13}$$

In the transient event described by equation (9), the variation of the voltage shows three different states according to the values of the roots.

1. No-vibration State

In equation (8), when $\frac{D^2}{4} > w_0^2$, both roots p_1 and p_2 are real, and both are negative. Variation of capacitor voltage and current with time:

$$U_C(t) = \frac{Ep_1p_2}{p_1 - p_2}(e^{p_1 t} + e^{p_2 t}) \tag{14}$$

$$I_C(t) = -C\frac{du_C}{dt} = -\frac{C \cdot E \cdot p_1 \cdot p_2}{p_2 - p_1}(e^{p_1 t} - e^{p_2 t}) \tag{15}$$

Since p_1 and p_2 are negative in equations (14) and (15), the voltage at the ends of the capacitor decreases exponentially. The variation of current and voltage is given in Figure 3.

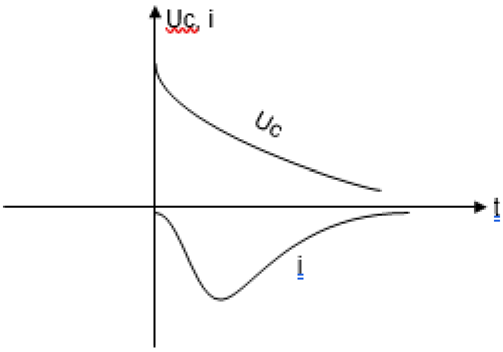

Figure 3

2. Vibration State

When $\frac{D^2}{4} < w_0^2$, roots are complex.

$$p_{1,2} = -\frac{D}{2} \pm j\sqrt{w_0^2 - \frac{D^2}{4}} = -\frac{D}{2} \pm jw_1 \tag{16}$$

The voltage change according to the values of these roots is obtained as in equation (17):

$$Uc(t) = e^{-\left(\frac{D}{2}\right)t}(k_1 \cos(w_1 t) + k_2 \sin(w_1 t)) \tag{17}$$

The current flowing through the circuit is obtained as in equation (18):

$$i(t) = A_1 \cos w_0 t \cdot e^{-\left(\frac{D}{2}\right)t} - A_2 \sin w_0 t \cdot e^{-\left(\frac{D}{2}\right)t} \tag{18}$$

The coefficients A_1 and A_2 are expressed as in equation (19):

$$A_1 = c_2 \cdot w_0 - c_1 \cdot D/2 \text{ and } A_2 = c_1 \cdot w_0 - c_2 \cdot D/2 \tag{19}$$

As seen in Figure 4, the variation of U_c is a sine vibration whose amplitude decreases with an exponential function ($e^{-(D/2)t}$) whose envelope is the sign function ($e^{-(D/2)t}$).

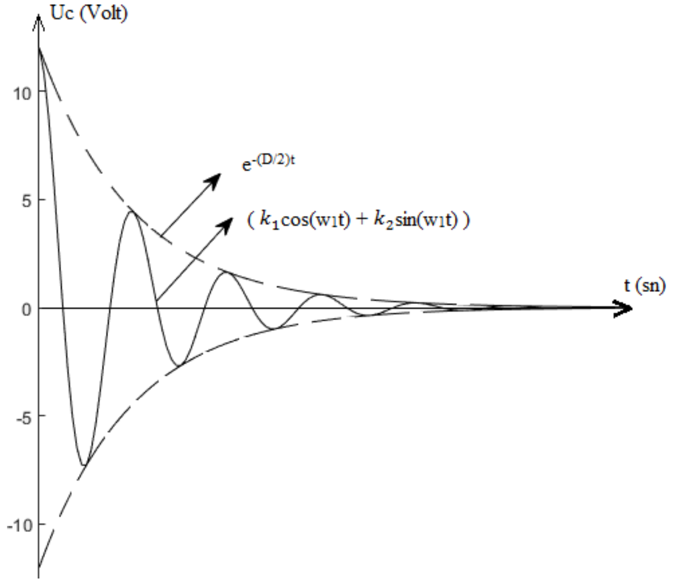

Figure 4

The general variation of the voltage was given by equation (17). The vibration frequency is smaller than the eigenfrequency w_0 of the circuit. The smaller the resistance determining the damping in the circuit, the smaller the difference between w_0 and w_1. If the resistance of the circuit were zero, the transient event would be a continuous sine vibration.

3. Vibration Boundary State

When $\frac{D^2}{4}=w_0^2$, $p_1=p_2= -D/2$. In this case, the change of voltage will be found in equation (20).

$$U_C(t) = k_1 \cdot e^{pt} + k_2 \cdot te^{pt} \tag{20}$$

In this case, where the roots are equal to each other, through using initial conditions, k_1=E and k_2= E. (D/2) is found, and the voltage change is obtained as in equation (21);

$$U_C(t) = E\left(1+\frac{D}{2}\cdot t\right)\cdot e^{-\left(\frac{D}{2}\right)t} \tag{21}$$

CONDUCTING THE EXPERIMENT

Materials Used in the Experiment

- DC source
- Time relay for switching
- 10 µF and 100 µF Capacitor
- 90 mH Inductance
- Adjustable resistor box
- Oscilloscope
- Connection cables

Set up the circuit given in Figure 5.

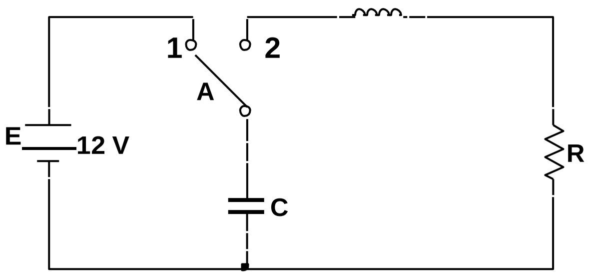

Figure 5

By turning switch A to position 1, capacitor C is charged with voltage E, and by turning the switch to position 2, a transient event is created in the circuit. The change in U_C voltage will be analyzed on the oscilloscope. Since the relay operates continuously, a stationary figure is obtained on the oscilloscope.

Experiment 1. Draw the variation of U_C voltage in the circuit with the help of an oscilloscope for R=0, 50, 100, 150, 190, 300, 1500, ∞ Ω values, respectively, on the graphs in the experiment report. Then, measure the duration of the transient (capacitor discharge time) with the oscilloscope and write these values in the tables in the experimental results section. Try to reconcile the changes you see with theoretical knowledge.

Experiment 2. The values of the elements used in the circuit are given as L=90 mH and C=10 µF. Calculate the critical resistance value required for the boundary state of vibration. Draw the U_C voltage change at this value on the graphs in the experiment report.

Experiment 3. Remove L from the circuit. With the help of an oscilloscope, plot the change of U_C voltage for R=0, 300, 500 Ω on the graphs in the experiment report. Measure the duration of the transient for each resistance value and record it in the table.